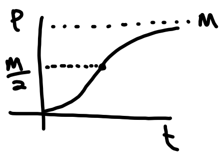
6.5 Logistic Growth

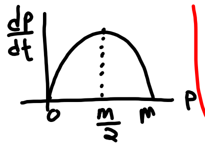
$$\boxed{\frac{dP}{dt} = kP(M-P)}$$

max Pop. (carrying capacity)

$\boxed{\text{as } P \to M,\ \frac{dP}{dt} \to 0}$

$\boxed{\lim_{t\to\infty} P(t) = M}$





$\boxed{\frac{dP}{dt} \text{ biggest at } P = \frac{M}{2}}$

Dec 7-7:35 PM

The growth rate of a population P of bears in a newly established wildlife preserve is modeled by the differential equation dP/dt = 0.008P(100-P), where t is measured in years.
(a) What is the carrying capacity for bears in this preserve?
(b) What is the bear population when the population is growing the fastest? P $\frac{dP}{dt}$ biggest
(c) What is the rate of change of the population when it is growing the fastest.

a) $M = 100$

b) 50

c) $\frac{dP}{dt} = .008(50)(100-50) = 20 \ \frac{\text{bears}}{\text{year}}$

Dec 7-8:03 PM

Sixty one moose were introduced to the upper peninsula in Michigan. The growth rate is given below. Solve for P

$$\frac{dP}{dt} = 0.0003P(1000-P)$$

$\int \frac{dP}{P(1000-P)} = \int 0.0003\, dt$ {separation of variables}

$\int \frac{1}{P(1000-P)}\, dP = \int 0.0003\, dt$

partial fractions

$\frac{1}{P(1000-P)} = \frac{A}{P} + \frac{B}{(1000-P)} = \frac{A(1000-P) + BP}{(1000-P)P}$

$1 = A(1000-P) + B \cdot P$

let $P = 0$ $\quad 1 = A \cdot 1000 \quad A = \frac{1}{1000}$

let $P = 1000$ $\quad 1 = B \cdot 1000 \quad B = \frac{1}{1000}$

$\frac{1}{P(1000-P)} = \frac{1/1000}{P} + \frac{1/1000}{1000-P} = \frac{1}{1000}\left(\frac{1}{P} + \frac{1}{1000-P}\right)$

Dec 7-8:16 PM

$\int \frac{dP}{P(1000-P)} = \int 0.0003\, dt$

$\frac{1}{1000}\int \frac{1}{P} + \frac{1}{1000-P}\, dP = \int 0.0003\, dt$

$-1000\left(\frac{1}{1000}\left(\ln P - \ln(1000-P)\right) = 0.0003t + c\right)$

$-\ln P + \ln(1000-P) = -0.3t + c$

$e^{\ln\left(\frac{1000-P}{P}\right)} = e^{(-0.3t + c)} = e^{-0.3t} \cdot e^{c}$

$\frac{1000-P}{P} = A e^{-0.3t} \cdot P + P$

$t = 0$ $\quad 1000 = P(Ae^{-0.3t} + 1)$

$P = 61$

$\frac{1000-P}{P} = Ae^{-0.3t} \quad P = \frac{1000}{1+Ae^{-0.3t}} \quad A = \frac{1000-61}{61}$

$P = \frac{1000}{1+15.4e^{-0.3t}} \quad A = 15.4$

Nov 29-9:52 AM

General Solution to the logistic equation

$$\boxed{\frac{dP}{dt} = kP(M-P)} \quad \boxed{P = \frac{M}{1+Ae^{-(Mk)t}} \qquad A = \frac{M-P_0}{P_0}}$$

Dec 7-8:28 PM

Partial Fractions

$$\frac{x-13}{2x^2-7x+3} = \frac{x-13}{(2x-1)(x-3)} = \frac{A}{2x-1} \cancel{\times} \frac{B}{x-3}$$

$$x-13 = A(x-3)+B(2x-1)$$

let $x=3$ $\quad -10 = 5B \quad B=-2$

let $x=\frac{1}{2}$ $\quad -12\frac{1}{2} = A(-2\frac{1}{2})$

$$-\frac{25}{2} = A\left(-\frac{5}{2}\right) \qquad A=5$$

$$\int \frac{x-13}{2x^2-7x+3}dx = \int \frac{5}{2x-1} - \frac{2}{x-3}\,dx$$

$$5\ln|2x-1|\cdot\frac{1}{2} - 2\ln|x-3| + C$$

Dec 7-8:09 PM

$$= kP(m-P)$$

$$\frac{dP}{dt} = 2P(1-.002P) = 2(.002)P(m-P)$$

what is $\lim_{t\to\infty} P(t)$? $= m$

$$\frac{dP}{dt} = .004P(500-P)$$

$$.002m = 1$$

$$m = \frac{1}{.002} = 500$$

Nov 29-10:20 AM